TITLE:

Nonlinear effects of the time dependent Gross-Pitaevskii equation in the potential scattering problem(3) Chaos and nonlinear dynamics in dissipative systems(including BEC and pattern formations), Chaos and Nonlinear Dynamics in Quantum-Mechanical and Macroscopic Systems)

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CITATION:

FUJISHIMA, Hironobu ...[et al]. Nonlinear effects of the time dependent Gross-Pitaevskii equation in the potential scattering problem(3) Chaos and nonlinear dynamics in dissipative systems(including BEC and pattern formations), Chaos and Nonline ...

ISSUE DATE:

2005-06-20

URL:

http://hdl.handle.net/2433/110222

RIGHT:

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Nonlinear effects of the time dependent Gross-Pitaevskii equation in the potential scattering problem

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中性原子ボーズ凝縮体のダイナミクスを支配すると考えられる時間依存 Gross-Pitaevskii 方程式 (非線形 Schrödinger 方程式) によるポテンシャル散乱問題を数値的に解析し, 具体的に斥力及び引力ポテンシャルの場合について波束の透過率及び反射率を調べる. その結果を通常の Schrödinger 方程式から得られる結果と比較すると, 非線形効果として (i) 粒子間斥力相互作用 ($g > 0$) が透過率を増加させること, (ii) 反射率は入射波束の速度やポテンシャルの形状に大きく影響され, その大小もそれらを変化させると入れ替わることなどがわかった.

1 The time dependent Gross-Pitaevskii equation

We investigate the dynamical behavior of wave packets in the potential scattering problem by numerically solving the time dependent nonlinear Schrödinger equation which is called the time dependent Gross-Pitaevskii equation (TDGPE). The TDGPE is widely regarded as the development equation for the order parameter of the Bose-Einstein Condensates, whose nonlinear term represents the binary interaction between the atoms. The TDGPE is

$$i\hbar\frac{\partial\psi(x,t)}{\partial t} = \left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x) + g|\psi(x,t)|^2\right)\psi(x,t),$$

where $V(x)$ is the box-type repulsive potential (BRP) or the well-type attractive potential (WAP) and the coupling constant g is expressed as $g = 4\pi\hbar^2 a_s/m$ by atom mass m and the s-wave scattering length a_s. Therefore, $g > 0$ means repulsive interaction between the atoms and $g < 0$ dose attractive interaction. For numerical calculation, we rewrite the TDGPE as the dimensionless form:

$$i\frac{\partial\tilde{\psi}(\xi,\tau)}{\partial\tau} = \left(-\frac{1}{2}\frac{\partial^2}{\partial\xi^2} + \tilde{V}(\xi) + \tilde{g}|\tilde{\psi}(\xi,\tau)|^2\right)\tilde{\psi}(\xi,\tau),$$

here we set the new parameters as $x = \sqrt{2}\hbar\xi/mv_0$, $t = 2\hbar\tau/mv_0^2$, $Ng = mv_0^2\tilde{g}/2$ and $\psi = \sqrt{N}\tilde{\psi}$, where N is the total number of condensed atoms and v_0 is typical incident velocity of the wave packet. Corresponding to the experiment at MIT [1], we fix the value of v_0 to be 3 mm/s. Hereafter, we omit tildes for brevity.

2 Results

In this section we briefly give the results of our numerical calculations. Throughout this section, we assume the wave packet approaches the potential from the right-hand-side. Fig.1 shows g-dependence of the transmission rate for the BRP defined as $\int |\psi(\xi,\tau)|^2 d\xi$ at sufficiently large τ where the integration area is the left-hand-side of the potential. Fig.1 also shows the reflection rate for the WAP defined similarly to the transmission case but integration is performed over the right-hand-side of the potential (In both cases the width of the potentials is choosen to be 2.0.). The increase of the transmission rate has been already reported in [2], however, we newly find that the transmission rate takes its minimum value when $g \simeq -2.0$ and that the reflection rate for the attractive potential increases monotonically as a function of g.

Fig.2 shows the incident velocity dependence of the reflection rate for the WAP. Reflection phenomena from attractive potentials manifest the wave-like nature of matter, MIT and JILA groups have successively performed relevant experiments with the Casimir-Polder attractive potential induced by a silicon surface [1]. Due to extremely low incident velocity required for the quantum reflection and finite size of the wave packet, time dependent treatment is indispensable. We find the role of g is rather complicated as shown in Fig.2, where the two lines which indicate the reflection rates with/without the repulsive inter-particle interactions cross each other as a function of the incident velocity. We also find the parameter region where the nonlinearity causes almost no effect on the reflection rate.

It is one of attractive future tasks to reveal the role of the nonlinearity over broader range of the parameters, which includes time dependent Fourier analysis of the wave number distribution and calculation with more realistic potential.

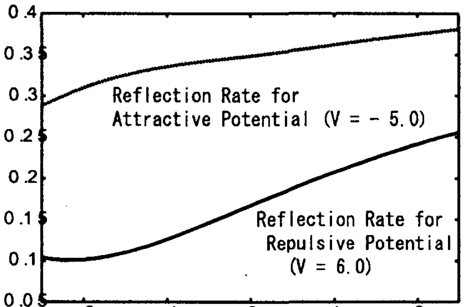

Fig. 1 g-dependence of Transmission and Reflection Rate

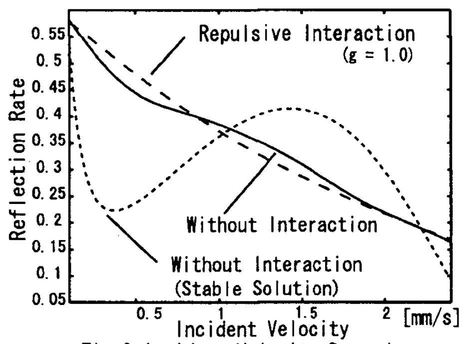

Fig. 2 Incident Velocity Dependence of Reflection Rate

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